Response of Orthogonally Stiffened Cylindrical Shell Panels

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STIFFENED shells find extensive use in the aircraft industry. The static analysis of cantilevered stiffened cylindrical shells subjected to internal pressure has been completed by Venkatesh and Rao,[1] using doubly curved anisotropic shell elements. The free vibration of stiffened cylindrical shells has been investigated experimentally by Hoppman[2] and analytically by Al-Najafi and Warburton,[3] using the finite-element method. Mustaffa and Ali[4] have studied the free vibration characteristics of stiffened cylindrical shells with shear diaphragm ends and panels that are cantilevered as well as free-free.

In this Note, the dynamic response of orthogonally stiffened cylindrical shell panels clamped on all edges subjected to a uniformly distributed step load has been considered. For the free vibration analysis, a series solution combined with Galerkin's method has been adopted. For the dynamic response, a normal mode method has been used.

Equations of Motion and Boundary Conditions

Using a smeared analysis for closely spaced stiffeners, the equations of motion for a stiffened cylindrical shell (Fig. 1) based on Donnell's theory and neglecting in-plane inertia can be derived (using the equations given in Ref. 5) in terms of the nondimensional variables as follows:

$$a_{11}\bar{u}_{,\bar{x}\bar{x}} + a_{12}\bar{u}_{,\bar{\theta}\bar{\theta}} + b_{11}\bar{v}_{,\bar{x}\bar{\theta}} + c_{11}\bar{w}_{,\bar{x}} + c_{12}\bar{w}_{,\bar{x}\bar{x}\bar{x}} = 0 \tag{1}$$

$$a_{21},\bar{u}_{,\bar{x}\bar{\theta}} + b_{21}\bar{v}_{,\bar{x}\bar{x}} + b_{22}\bar{v}_{,\bar{\theta}\bar{\theta}} + c_{21}\bar{w}_{,\bar{\theta}} + c_{22}\bar{w}_{,\bar{\theta}\bar{\theta}\bar{\theta}} = 0 \tag{2}$$

$$a_{31}\bar{u}_{,\bar{x}\bar{x}\bar{x}} + a_{32}\bar{u}_{,\bar{x}} + b_{31}\bar{v}_{,\bar{\theta}} + b_{32}\bar{v}_{,\bar{\theta}\bar{\theta}\bar{\theta}} + c_{31}\bar{w} + c_{32}\bar{w}_{,\bar{x}\bar{x}\bar{x}\bar{x}}$$
$$+ c_{33}\bar{w}_{,\bar{x}\bar{x}\bar{\theta}\bar{\theta}} + c_{34}\bar{w}_{,\bar{\theta}\bar{\theta}\bar{\theta}\bar{\theta}} + c_{35}\bar{w}_{,\bar{\theta}\bar{\theta}} + c_{36}\bar{w} = \bar{Q} \tag{3}$$

where

$$\bar{u} = (a/h^2)u, \qquad \bar{v} = (a/h^2)v, \qquad \bar{w} = (w/h)$$

$$\bar{x} = (x/a), \qquad \bar{\theta} = (\theta/\theta_0), \qquad \bar{Q} = QR/Eh$$

and where Q is the uniformly distributed loading.

The panel is clamped on all edges. Then the boundary conditions are

$$\bar{u} = \bar{v} = \bar{w} = \bar{w}_{,\bar{x}} = 0 \qquad \text{along } x = 0 \text{ and } 1 \tag{4}$$

$$\bar{u} = \bar{v} = \bar{w} = \bar{w}_{,\bar{\theta}} = 0 \qquad \text{along } \bar{\theta} = 0 \text{ and } 1 \tag{5}$$

Solution

Displacements can be expressed as follows:

$$\bar{u} = \sum_m \sum_n A_{mn}\phi_m(\bar{x})Y_n(\bar{\theta}) \tag{6}$$

$$\bar{v} = \sum_m \sum_n B_{mn}X_m(\bar{x})\psi_n(\bar{\theta}) \tag{7}$$

$$\bar{w} = \sum_m \sum_n C_{mn}X_m(\bar{x})Y_n(\bar{\theta}) \tag{8}$$

in which X_m, Y_n, ϕ_m, ψ_n correspond to the free-vibration mode shapes of beams with different boundary conditions.

That is

$\phi_m(\bar{x})$, $\psi_n(\bar{\theta})$ both ends simply supported

$X_m(\bar{x})$, $Y_n(\bar{\theta})$ both ends clamped

Substituting Eqs. (6-8) into Eqs. (1-3) and using Galerkin's procedure, one gets three equations in terms of A_{mn}, B_{mn}, and C_{mn}. With A_{mn} and B_{mn} eliminated, the equations are solved for dynamic response.

Convergence

A convergence study with respect to the number of terms ($m = n = 3$, 5, 7) in Eqs. (6-8) has been conducted for the following shell panel considering nine normal modes.

Young's modulus = 7.1148×10^6 N/cm^2

Poisson's ratio = 0.22

Density = 2.7×10^{-6} Ns^2/cm^4

$R = 100$ cm, $a = 100$ cm, $h = 0.5$ cm

$\theta_0 = 45$ deg, $B_\alpha = B_\theta = 0.5$ cm, $D_\alpha = D_\theta = 2.0$ cm

$b_\theta = 10$ cm, $\hat{\theta} = 5.73$ deg

The variation of central deflection is presented in Fig. 2. It is observed that there is good convergence. Hence, for further study, seven terms ($m = n = 7$) were taken. Later, after a convergence study for normal modes, it was decided to take 10 modes for further work.

A study was conducted on the dynamic response of stiffened panel by changing the ratio of volume of material in skin to that in stiffeners. Total volume was kept constant. Spacing

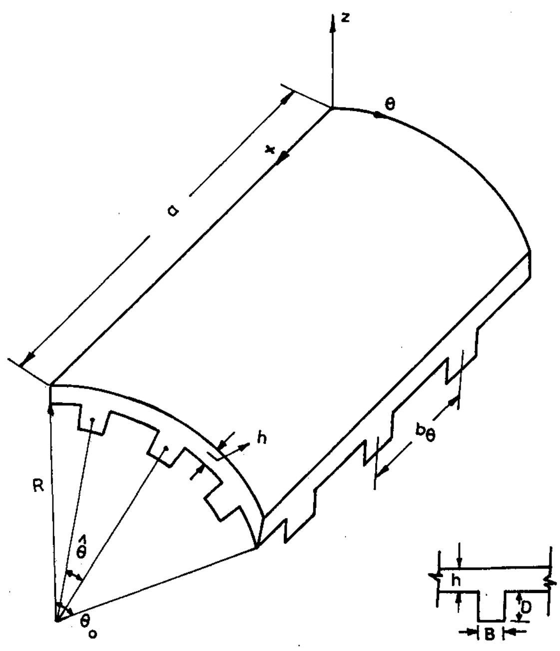

Fig. 1 Stiffened cylindrical shell.

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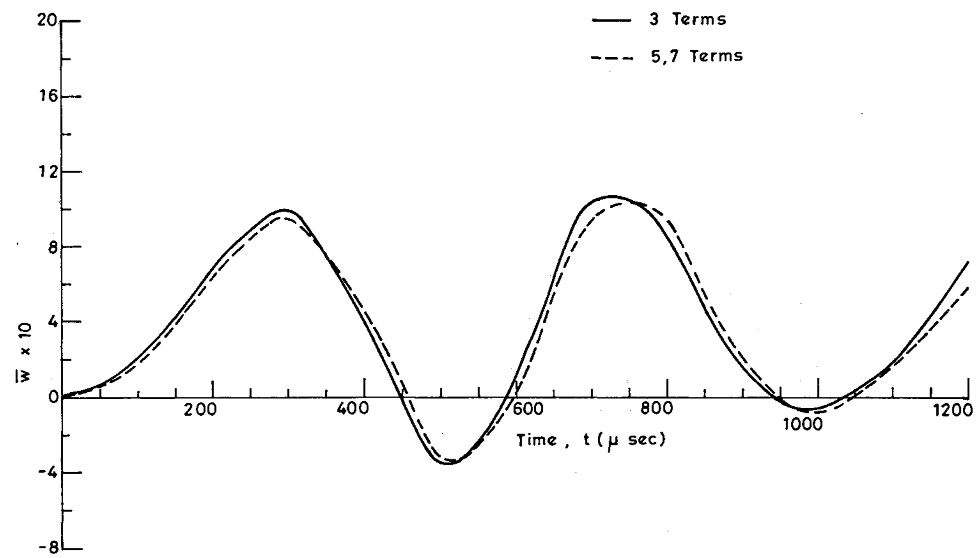

Fig. 2 Convergence of central deflection.

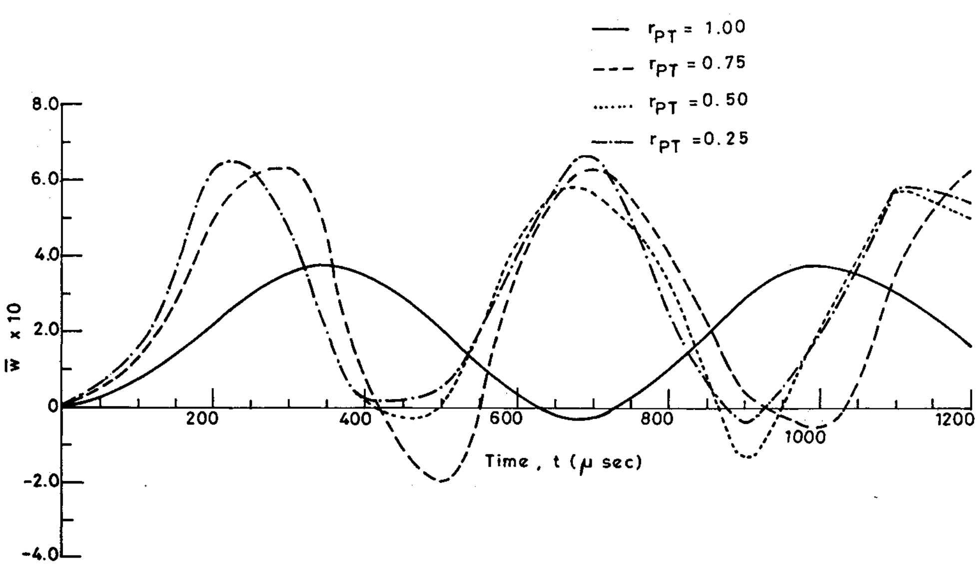

Fig. 3 Central deflection vs time.

Table 1 Details of stiffeners

$R = 100$ cm,	$a = 100$ cm,	$\theta_0 = 45$ deg,	$b_\theta = 10$ cm,	$\theta = 5.73$ deg
r_{pT},[a] cm	1.0	0.75	0.50	0.25
B_α	0	0.591	0.835	1.023
D_α	0	2.363	3.341	4.092
B_θ	0	0.591	0.835	1.023
D_θ	0	2.363	3.341	4.092

[a]Where $r_{pT} = \dfrac{\text{volume of material in skin}}{\text{total volume of material in both stiffeners and skin}}$

of stiffeners, as well as breadth-to-depth ratio of stiffeners, was kept constant. The elastic properties of skin and stiffeners were the same as those taken for the convergence study. Geometric properties for the four cases studied are given in Table 1.

The central deflection (in nondimensional form) for various cases is presented in Fig. 3. From the figure, it may be observed that deflection decreases as r_{pT} decreases.

References

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